Homework 1 – Sketch of Solutions

1.2 $4f: -x^3 + 7x^2 + 4$

$8: \quad (a+b)(x+y) = (a+b)x + (a+b)y$
$= ax + bx + ay + by$
$= ax + ay + bx + by$

10: The sum of two diff. functions is diff. A scalar (element of $\mathbb{R}$) times a diff. function is diff. Both statements are proved in calculus

12: The sum of two even functions $f(t)$ and $g(t)$ is $f(t) + g(t)$. Then

$$f(-t) + g(-t) = f(t) + g(t)$$

and $(af)(-t) = af(-t) = af(t) = (af)(t)$

15: No. For example if $x = (a_1, a_2, \dots, a_n) \neq 0$ in $\mathbb{R}^n$
$ix = (ia_1, \dots, ia_n)$ which is not in $\mathbb{R}^n$

21: $((v,w) + (v',w')) + (v'',w'') = ((v+v')+v'', (w+w')+w'')$
and $(v,w) + ((v',w') + (v'',w'')) = (v + (v'+v''), w + (w'+w''))$
But $(v+v')+v'' = v + (v'+v'')$ and $(w+w')+w'' = w + (w'+w'')$

1.3 2d:

$$A^t = \begin{pmatrix} 10 & 2 & -5 \\ 0 & -4 & 7 \\ -8 & 3 & 6 \end{pmatrix} \qquad \operatorname{tr} A = 10 - 4 + 6 = 12$$

5: $(A + A^t)^t = A^t + (A^t)^t = A^t + A = A + A^t$

10: Let $x = (a_1, \dots, a_n)$, $y = (b_1, \dots, b_n) \in W_1$ and $a \in F$
$x + y = (a_1 + b_1, \dots, a_n + b_n)$ and $(a_1 + b_1) + \dots + (a_n + b_n) =$
$(a_1 + \dots + a_n) + (b_1 + \dots + b_n) = 0 + 0 = 0$
$ax = (aa_1, \dots, aa_n)$ and $aa_1 + \dots + aa_n = a(a_1 + \dots + a_n) = a0 = 0$

Also $0 = (0,0,\dots,0) \in W_1$ $\therefore W_1$ is subspace

Note $0 \notin W_2$ $\therefore W_2$ is not a subspace

11: No. The sum of two polynomials of degree n could have degree $< n$ (find an example)

20: $w_1, w_2, \dots, w_n \in W$; $a_1, \dots, a_n \in F$

$\therefore a_1 w_1, a_2 w_2, \dots, a_n w_n \in W$ (property of subspaces)

$\therefore a_1 w_1 + a_2 w_2 \in W$

$\therefore (a_1 w_1 + a_2 w_2) + a_3 w_3 \in W$

$\vdots$

$(a_1 w_1 + \cdots + a_{n-1} w_{n-1}) + a_n w_n \in W$

$\therefore a_1 w_1 + \cdots + a_n w_n \in W$

23: Let U be a subspace containing W_1, W_2. Let $w_1 + w_2 \in W_1 + W_2$ $\quad w_1 \in W_1 \subseteq U$, $w_2 \in W_2 \subseteq U$

$\therefore w_1 + w_2 \in U$ $\quad \therefore W_1 + W_2 \subseteq U$

25: Clearly $W_1 \cap W_2 = \{0\}$

Let $h(x) = a_0 + a_1 x + \cdots + a_m x^m$

$f(x) = a_0 + a_2 x^2 + a_4 x^4 + \cdots$

$g(x) = a_1 x + a_3 x^3 + a_5 x^5 + \cdots$

$\therefore h(x) = f(x) + g(x)$ so $V = W_1 \oplus W_2$

1.4 6: Given any vector $(a,b,c) \in F^3$ find scalars x, y, z such that

$$(a,b,c) = x(1,1,0) + y(1,0,1) + z(0,1,1)$$

$$\begin{aligned} x+y&=a \\ x+z&=b \\ y+z&=c \end{aligned} \rightsquigarrow \begin{aligned} x+y&=a \\ y+z&=c \\ -y+z&=b-a \end{aligned} \rightsquigarrow \begin{aligned} -z&=a-c \\ y+z&=c \\ z&=\frac{b-a+c}{2} \end{aligned}$$

$\therefore x = a-c+\frac{b-a+c}{2}$, $y = c - \frac{b-a+c}{2}$, $z = \frac{b-a+c}{2}$

$\underline{10}$: Let W be the span

$A \in W \iff A = a\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + c\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ for

some scalars a, b, c $= \begin{pmatrix} a & c \\ c & b \end{pmatrix}$ which is a typical symmetric

matrix.